

# On the Optimal Timing of Foreign Trade

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## **Abstract**

In this paper, we consider a two-sector, dynamic model of an economy in which one sector grows relative to another one, causing a declining of its relative price. We examine how trade may affect production, consumption, pattern of trade, and welfare of the economy. We argue that if the timing of trade can be chosen, free trade is always better than no trade. In some cases, it pays to subsidize manufacturing production and to promote industrialization.

# 1 Introduction

One of the oldest issues in the theory of international trade is how foreign trade may affect the welfare of an economy. The traditional analysis focuses on the static gains from trade for economies with fixed technologies, preferences, and factor endowments.<sup>1</sup>

Recently, there has been growing interest in examining the welfare impacts of trade in a dynamic context. For example, Kemp and Long (1979), Binh (1985), Serra (1991), and Kemp and Wong (1995) analyze how trade may affect the intertemporal welfare of economies with overlapping generations,<sup>2</sup> while Grossman and Helpman (1991a), Baldwin (1992), and Taylor (1994) examine how the welfare of economies with endogenous growth may be affected by trade. These papers, however, share one common feature: they all consider models that are characterized by constant relative prices in steady states.<sup>3</sup> What this implies is that as far as steady states (if exist) are concerned, the results are time invariant. Let us call this type of economies *FRP economies*, i.e., those that experience *fixed relative prices* in steady states.

Another class of dynamic models has been introduced to examine the dynamic impacts of foreign trade. For example, Feenstra (1996), extending the two-country model of Grossman and Helpman (1991b), explains how different rates of innovation in different sectors may cause persistent changing relative prices and how international trade may lead to uneven growth performance across countries. Similar ideas are also developed by Young (1991) using a model of learning by doing. These models describe *CRP* (*changing relative prices* in steady states) economies, in which relative prices change over time

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<sup>1</sup>The literature on gains from trade is huge. A recent survey and some extensions are given in Wong (1995, Chapters 8 and 9).

<sup>2</sup>While examples have been constructed to show that with overlapping generations uncompensated trade is Pareto inferior to autarky, Kemp and Wong (1995) argue that there are four compensation schemes that a government may use to ensure that trade is Pareto improving.

<sup>3</sup>It is common in the growth literature to postulate constant relative prices in the steady states. See Bond, Wang, and Yip (1997) for a general characterization of this class of growth models.

in steady states. The existence of CRP economies is consistent with the fact that for many countries some commodities are getting continuously cheaper and cheaper relative to others over a long period of time.

Welfare and policy analysis is important for CRP economies. Since production and consumption decisions respond to relative prices in competitive markets, the changes in resource allocation and welfare in steady states could be quite different for CRP economies than for FRP economies. Furthermore, the impacts of foreign trade and those of trade policies could depend on when foreign trade is allowed or when a trade policy is imposed.

However, to the best of our knowledge, there has not been any paper in the literature that analyzes the dynamic gains from trade for CRP economies. It is thus the objective of this paper to fill this gap in the literature. We consider a two-sector economy in which one of the sectors (manufacturing) grows over time with respect to the other one (agriculture) due to accumulation of human capital through learning by doing. The growth of manufacturing leads not only to growth of the economy but also to a declining relative price of manufacturing.<sup>4</sup>

We examine the features of trade between this economy and the rest of the world, when both are growing and experiencing declining relative price of manufacturing over time. Several different cases can be identified, depending on the initial comparative advantage of the economy and the growth rates of the economy and the rest of the world. Using these cases, we can try to analyze several questions. First, is the initial comparative advantage of the economy sustainable over time? Would there be reversal of pattern of trade? Second, if free trade is allowed initially, would it be gainful in a dynamic sense? Would free trade contribute to an increase in the intertemporal welfare of the economy? Third, if the timing of free trade can be chosen by the government, would free trade be gainful dynamically? Does it make sense for the government to delay free trade? Fourth, what is the optimal trade policy of the government? If production subsidy is permitted, would the optimal production subsidy be non-zero?

These questions surround the major issue we are looking at in this paper: the timing of free trade. As a matter of fact, in the present model, the initial comparative advantage may not be sustained. The model also shows that it

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<sup>4</sup>Our model can be justified by the fact that some manufacturing goods are getting cheaper relative to many other products over time; for example, computers, TV sets, cameras, and so on.

is probably not enough to say whether free trade should be allowed, but also when free trade is allowed.<sup>5</sup> Our analysis shows that free trade starting from the beginning is not necessary gainful dynamically. This is not surprising because of the existence of domestic distortion: dynamic externality. What is surprising is that if the government chooses the timing of free trade optimally, free trade is always good, i.e., no trade is never the optimal policy. In this paper we also examine whether it makes sense to impose a production subsidy to try to correct the dynamic externality. We show that such a policy is sometimes helpful, but it is also shown that there are cases in which free trade is enough to remove the distortion caused by dynamic externality so that no production subsidy is necessary.

This paper is organized as follows. In Section 2, we describe the closed economy. With external learning by doing as a distortion, policies like a production subsidy will improve the economy's lifetime welfare and we derive the optimal production subsidy formula. Section 3 analyzes free trade and the pattern of production of the economy. Section 4 examines the dynamic gains from free trade. Section 5 investigates the optimal timing of trade, with and without a suitably chosen production subsidy. A discussion of how the present analysis with the well-known infant-industry argument for protection is also provided. The last section concludes.

## 2 The Closed Economy

Consider a two-sector, dynamic economy. Two homogeneous consumption goods, which for convenience are labeled agriculture and manufacturing, are produced by competitive firms.<sup>6</sup>

### 2.1 Technology

The production of agriculture (good A) requires only labor input, and its sectoral production function can be written as:

$$X_t^A = AL_t^A, \tag{1}$$

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<sup>5</sup>Our way of posing this question includes the possibility of having no trade as the optimal policy because autarky is equivalent to allowing free trade in infinite.

<sup>6</sup>Our closed economy is similar to the Ricardo-Viner model in Matsuyama (1992).

where  $X_t^A$  is the agriculture output,  $L_t^A$  is the labor input at any time  $t \in [0, \infty]$ , and  $A > 0$  denotes the constant labor productivity. Since  $A$  is constant, it is equal to the marginal product as well as average product of labor of the sector.

Production of manufacturing (good M) requires two inputs: labor ( $L^M$ ) and an intangible capital ( $M$ ),

$$X_t^M = F(M_t, L_t^M), \quad (2)$$

where  $X_t^M$  is the manufacturing output. The intangible capital mimics the concept of “experience” or “knowledge” in production. While it is taken by the firms as constant at any time  $t$ , it increases over time according to the following costless learning-by-doing process:

$$\dot{M}_t = \mu X_t^M = \mu F(M_t, L_t^M), \quad (3)$$

where  $\mu > 0$  is a measure of the effectiveness of learning by doing and a dot above a variable means its time derivative.<sup>7</sup> By condition (3),  $M_t$  is the engine of growth in the model. We assume that the initial value of the intangible capital,  $M_0$ , is given and that  $M_t$  does not depreciate. More specifically, we assume that the production function  $F(., .)$  is subject to constant returns in  $M_t$  and takes the following form:

$$F(M_t, L_t^M) = BM_t L_t^M, \quad (4)$$

where  $B > 0$  is the technology index, which is constant over time. Firms take  $M_t$  at any time as given, perceiving that their output level is proportional to labor employment. Choosing agriculture as the numeraire, we denote the relative consumers price of manufacturing by  $p_t$ . In addition, we consider a production subsidy of constant ad valorem rate of  $s > -1$  on the manufacturing sector so that the domestic manufacturing price faced by producers becomes  $(1 + s)p_t$ .<sup>8</sup> Perfect and costless mobility of labor between the two sectors with positive outputs implies equalization of wage rates:

$$A = (1 + s)p_t B M_t. \quad (5)$$

For simplicity, we assume that the economy is endowed with a constant labor force,  $L$ .

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<sup>7</sup>We do not consider depreciation of knowledge capital. See also Matsuyama (1992)

<sup>8</sup>If  $s < 0$ , it is a production tax. If  $s = 0$ , consumers prices are equal to producers prices.

## 2.2 Preferences

Assume that the instantaneous utility function of a representative agent at time  $t$  is given by  $\beta \ln C_t^A + \ln C_t^M$ , where  $C_t^i$  is the consumption of good  $i$  at time  $t$ ,  $i = A, M$ , and  $\beta > 0$ . The optimization problem of the representative agent is to choose the consumption stream to maximize lifetime welfare,

$$W = \max \int_0^{\infty} (\beta \ln C_t^A + \ln C_t^M) e^{-\rho t} dt, \quad (6)$$

subject to a standard budget constraint

$$C_t^A + p_t C_t^M = AL_t^A + (1 + s)p_t BM_t L_t^M - T_t, \quad (7)$$

as well as (3), where  $\rho$  is the rate of time preferences and  $T_t$ , treated as constant by the agent, denotes the lump-sum tax used to finance the production subsidy. Letting  $\lambda_t$  be the costate variable associated with (3), the first-order conditions for the optimization problem are

$$\beta p_t / C_t^A = 1 / C_t^M \quad (8)$$

$$\dot{\lambda}_t = \rho \lambda_t - \lambda_t \mu BL_t^M - \beta(1 + s)p_t BL_t^M / C_t^A, \quad (9)$$

as well as (3), (7) and the transversality condition. Given a Cobb-Douglas type utility function, the representative agent chooses to consume both goods at all finite, positive prices.

## 2.3 Closed-economy Equilibrium

We now derived the closed-economy (autarkic) equilibrium of the model. Equilibrium of the two commodity markets is given by

$$C_t^M = BM_t L_t^M, \quad (10)$$

$$C_t^A = AL_t^A. \quad (11)$$

Equilibrium of the labor market is

$$L_t^A + L_t^M = L. \quad (12)$$

By making use of the production functions (1) and (4), and the equilibrium condition (12), the production possibility frontier (PPF) of the economy at time  $t$  is described by the following equation:

$$X_t^A = AL - \frac{A}{BM_t} X_t^M. \quad (13)$$

The marginal rate of transformation (MRT) of the economy, denoted by  $q_t$ , is equal to the magnitude of the slope of the PPF, or, by (13), equal to

$$q_t \equiv -\frac{dX_t^A}{dX_t^M} = \frac{A}{BM_t}. \quad (14)$$

Because the intangible capital  $M_t$  is growing over time, the MRT is declining at the same rate. By condition (5), the producers' price ratio is equal to the MRT, i.e.,  $q_t = (1 + s)p_t$ .

The optimality and equilibrium conditions can be rewritten as follows:

$$C_t^A = \beta p_t C_t^M \quad (15)$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \mu B L_t^M - \frac{1 + s}{\lambda_t M_t} \quad (16)$$

$$\dot{M}_t = \mu B M_t L_t^M, \quad (17)$$

as well as (5) and (10)–(12). Combining (5), (10)–(12), and (15) yields

$$L^M = \frac{(1 + s)L}{1 + s + \beta}. \quad (18)$$

Condition (18) has four implications. First,  $L^M \in (0, L)$  for any finite  $s > -1$ , meaning that the economy is diversified. Second, the equilibrium value of  $L^M$  is independent of prices. Third,  $L^M$  is constant over time in equilibrium. This further implies that  $L^A$  and thus consumption and production of agriculture is constant in equilibrium. Fourth, a rise in  $s$  increases  $L^M$ :

$$\frac{dL^M}{ds} = \frac{\beta L}{(1 + s + \beta)^2} > 0, \quad (19)$$

i.e., an increase in  $s$  induces more labor from the agricultural sector to the manufacturing sector, encouraging the production of the latter.

## 2.4 Balanced Growth Path

The balanced growth path (BGP) equilibrium of the economy is defined as a situation in which all endogenous variables are changing at constant rates (not necessarily the same). Based on this definition, the autarkic BGP equilibrium of the economy is described by the following proposition:



**Proposition 1** *The autarkic BGP equilibrium of the economy, under any given subsidy rate  $s$ , is a situation in which  $C_t^M$ ,  $X_t^M$ , and  $M_t$  ( $\lambda_t$  and  $p_t$ ) are growing (declining) at a common constant rate of  $g^a$  while  $C_t^A$ ,  $X_t^A$ ,  $L_t^A$ , and  $L_t^M$  are stationary over time.*

**Proof.** By condition (18),  $L_t^M$  is constant over time. Let the BGP equilibrium growth rate of  $M_t$  be  $g^a$ . Then (5) yields  $\dot{p}_t/p_t = -g^a$ . Next, (11) and (1) give constancy of  $L_t^A$ ,  $X_t^A$  and  $C_t^A$ , while (15) in turn implies that  $C_t^M$  (hence  $X_t^M$ ) is growing at the same rate  $g^a$ . Finally, condition (16) implies that  $\lambda_t$  is declining at the same rate  $g^a$ . ■

Using Proposition 1, we can derive the BGP growth rate. Imposing the BGP equilibrium restrictions on (15) – (17), we get

$$C^{Aa} = \beta p_t^a C_t^M \quad (20)$$

$$-g^a = \rho - \mu B L^M - (1 + s)/M_t^a \lambda_t^a \quad (21)$$

$$g^a = \mu B L^M, \quad (22)$$

where the superscript “ $a$ ” is used to denote the autarkic BGP value of a variable. By Proposition 1,  $C^{Aa}$  in (20) is constant over time, where from (11), (12), and (18), we have

$$C^{Aa} = \frac{\beta A L}{1 + s + \beta}. \quad (23)$$

Substituting (18) into (22) to yield

$$g^a = \frac{(1 + s)\mu B L}{1 + s + \beta}, \quad (24)$$

which implies that the autarkic growth rate depends on  $B$ ,  $\mu$ , and  $s$ . We are particularly interested in the effect of a production subsidy on the growth rate. Let us write  $g^a \equiv g^a(s)$ . Without production subsidy, the growth rate of the economy is  $g^a(0) \equiv \mu B L / (1 + \beta)$ .

Condition (22) means that the autarkic BGP growth rate is proportional to the manufacturing employment, and thus output level, implying that the maximum growth rate of this economy is equal to

$$\bar{g} = \mu B L, \quad (25)$$

when the economy is completely specialized in manufacturing.

The effects of the subsidy on the autarkic growth rate are shown by the following derivatives:

$$\frac{\partial g^a}{\partial s} = \frac{\mu\beta BL}{(1+s+\beta)^2} > 0 \quad (26)$$

$$\frac{\partial^2 g^a}{\partial s^2} = -\frac{\mu\beta BL}{(1+s+\beta)^3} < 0. \quad (27)$$

Conditions (26) and (27) imply that  $g^a$  is strictly increasing and strictly concave in  $s$ . Based on these two conditions, the dependence of the growth rate on  $s$  is illustrated by schedule GG in Figure 1. It is clear from condition (24) that  $g^a$  is bounded from above by  $\bar{g} \equiv \mu BL$ , but it is approaching  $\bar{g}$  as  $s$  approaches infinity.

By condition (5), the term  $p_t M_t$  is constant. Since  $M_0$  is given, it is required that the initial autarkic price ratio,  $p_0^a$ , has to adjust instantaneously to satisfy the labor mobility condition, i.e.,

$$p_0^a = \frac{A}{(1+s)BM_0}. \quad (28)$$

Condition (28) suggests that when given  $M_0$ , an increase in  $s$  lowers the required initial price level.

To close this subsection, we briefly discuss the transitional dynamics of the closed economy. Defining  $m_t = M_t \lambda_t$  and using (16) and (17), we can summarize the dynamics in the following linear autonomous ordinary differential equation:

$$\dot{m}_t = \rho m_t - (1+s). \quad (29)$$

We illustrate (29) in Figure 2, which highlights the fact that  $m_t$  is unstable. Thus, there cannot be any transition in terms of  $m_t$  in the closed economy and the BGP equilibrium must be achieved through an instantaneous adjustment of the shadow price of knowledge.

## 2.5 Welfare

With no transition in our closed economy, we can study the welfare consequence of the production subsidy by focusing exclusively on the BGP equilibrium. In this subsection, we derive the formula for the optimal production subsidy.

Noting the absence of transition, condition (6) can be simplified to give the autarkic lifetime welfare

$$W^a = \frac{1}{\rho} [(1 + \beta) \ln C^{Aa} - \ln \beta - \ln p_0^a] + \frac{g^a}{\rho^2}, \quad (30)$$

where condition (23) and the fact that  $p_t$  is declining at the rate of  $g^a$  along a BGP have been used. By condition (30), and using (18), (23), and (28), an increase in  $s$  affects the autarkic welfare through three channels: a drop in  $C^{Aa}$ , a drop in  $p_0^a$ , and a rise in  $g^a$ . The first channel leads to a negative effect while the other two produce positive effects. Differentiate (30) with respect to  $s$  to give

$$\frac{dW}{ds} = \frac{\mu\beta BL}{\rho^2(1+s+\beta)^2} - \frac{s\beta}{\rho(1+s)(1+s+\beta)}, \quad (31)$$

which in general has an ambiguous sign. However, condition (31) shows that if  $s$  is zero or sufficiently small, the derivative is positive, implying that a small production subsidy is welfare improving. If  $s$  is sufficiently large,  $C^{Aa}$  is so small that  $\ln C^{Aa}$  is very negative, leading to a welfare below the level with no intervention. We thus conclude that a positive, optimal subsidy,  $s^{a*}$ , exists, and is obtained by setting  $dW^a/ds = 0$  and rearranging the terms to obtain

$$s^{a*} = \frac{g^a(s^{a*})}{\rho}, \quad (32)$$

where  $g^a(s)$  is given by (24). Conditions (32) and (24) can be combined to give the optimal production subsidy. Graphically, this is the intersecting point, denoted by point S, between schedule GG and a ray with a slope of  $\rho$  in Figure 1. Since schedule GG is positively sloped and concave, with a vertical intercept of  $g^a(0) > 0$ , the optimal subsidy exists and is unique, with the economy remaining diversified.

Finally, it is straightforward to see from (24) and (32) that the optimal subsidy is decreasing in both  $\beta$  and  $\rho$ , but increasing in  $\mu$ ,  $B$  and  $L$ .

### 3 Free Trade and Production Patterns

Let the economy introduced above be now called the home economy, which is able to trade with the rest of the world (ROW). To simplify our analysis, the following assumptions are made: (a) Home is small as compared with the

ROW in the sense that the economic conditions in the ROW are not affected by its trade with the economy. (b) The structure of the ROW is the same as the home economy. (c) At the time when trade is allowed, both the economy and the ROW are at their own BGP equilibrium. (d) For the time being, no production subsidy by any country is considered both before and after trade. (e) There is no international spillover of knowledge, meaning that the home economy learns from its own manufacturing production only.

Denote the exogenously given BGP growth rate of the ROW by  $g^w > 0$ , and the relative price of manufacturing in the ROW at time  $t$  by  $p_t^w > 0$ , which is decreasing at a rate of  $g^w$ .

For the time being, we consider the case in which free trade exists for  $t \geq 0$ . The cases in which free trade is possibly allowed some time in the future will be analyzed later. Let us make the following definitions.

- *Potential Comparative Advantage* — This is the comparative advantage of the economy should free trade not be allowed for  $t \geq 0$ . It is determined by comparing the MRT of the economy under autarky at time  $t$ ,  $q_t^a$ , with the prevailing world price ratio,  $p_t^w$ . The potential comparative advantage is used to find what the economy would export at  $t = t' \geq 0$  if the economy is under autarky for  $t < t'$  but free trade is allowed at  $t = t'$ .
- *Actual Comparative Advantage* — This is the comparative advantage of the economy in the presence of free trade. It is determined by comparing its actual MRT under free trade,  $q_t^f$ , with the prevailing world price ratio,  $p_t^w$ . The actual comparative advantage dictates what the economy is actually exporting.

We thus say that an economy is exporting the right commodity at  $t = t' \geq 0$  if its actual exportable is consistent with its potential comparative advantage. With a Ricardo-type technology, the economy (a) is completely specialized in and exports agriculture at time  $t$  if

$$p_t^w < q_t^f, \quad (33)$$

or (b) is completely specialized in and exports manufacturing if

$$p_t^w > q_t^f. \quad (34)$$

Note that because free trade is allowed at  $t = 0$ ,  $q_0^a \equiv q_0^f$ . These two cases of specialization are analyzed separately as follows.

### 3.1 Specialization in Agriculture: Case SA<sub>0</sub>

If condition (33) is satisfied at  $t = 0$ , i.e.,  $p_0^w < q_0^a$ , then the economy will be immediately specialized in agriculture under free trade. Let us call this case SA<sub>0</sub>.<sup>9</sup> With no production of manufacturing, there is no learning by doing, meaning that the intangible capital stays constant at its initial level,  $M_0$ , and that the production possibility frontier remains stationary. Because the world price is declining over time, condition (33) is always satisfied, implying that specialization in agriculture is sustained.

We now derive the BGP of the economy. With production of agriculture only, the national income in terms of agriculture is constant and equal to  $AL$ . With the Cobb-Douglas type utility function, the optimal consumption allocations are:

$$C_t^A = \left( \frac{\beta}{1 + \beta} \right) AL \quad (35)$$

$$p_t^w C_t^M = \left( \frac{1}{1 + \beta} \right) AL. \quad (36)$$

Thus, the home economy exports  $AL/(1 + \beta)$  units of agriculture to the rest of the world in exchange for the equivalent value of manufacturing at the world price  $p_t^w$ . Since the world price is falling, the economy experiences improving terms of trade, and the quantity of manufacturing it imports is growing over time. We summarize the characterization of the BGP equilibrium in the following proposition:

**Proposition 2** *Suppose that  $p_0^w < q_0^a$  and that free trade is allowed for  $t \geq 0$ . (a) The economy is completely specialized in agriculture. This production pattern is sustainable. (b) The free-trade BGP with specialization in agriculture is a situation where  $C_t^M$  is growing at the given rate of  $g^w$  while  $C_t^A$  and  $X_t^A$  are stationary over time. The home economy exports agriculture of the amount of  $AL/(1 + \beta)$  and imports an equal value of manufacturing.*

### 3.2 Specialization in Manufacturing: SM<sub>0</sub>

Consider now the case in which  $p_0^w > q_0^a$ . If free trade is allowed for all  $t \geq 0$ , the home economy is completely specialized in manufacturing. Call this case

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<sup>9</sup>The terminology adopted in this paper is: SA<sub>0</sub> (SM<sub>0</sub>)  $\equiv$  specialization in agriculture (manufacturing) when free trade is allowed from  $t = 0$ .

SM<sub>0</sub>. With all of its labor allocated to the manufacturing sector, the economy achieves its maximum growth rate,  $\bar{g} = \mu BL$ , which is the growth rate of the intangible capital. By (14), the MRT of the economy,  $q_t^f$ , is declining at the rate of  $\bar{g}$ . Depending on the values of  $\bar{g}$  and  $g^w$ , two sub-cases can be identified:

### 3.2.1 The Strong SM<sub>0</sub> Case: $\bar{g} \geq g^w$

In this sub-case, because the economy is growing at a rate faster than that of the ROW, the economy's MRT  $q_t^f$  is always less than the world price ratio  $p_t^w$ , implying that condition (34) is always satisfied and the pattern of production is sustained. As a result, the national income at any time is given by  $p_t^w BM_t L$  (in terms of agriculture), which is increasing at the rate of  $\bar{g} - g^w$ . The optimal consumption allocations are:<sup>10</sup>

$$C_t^A = \left( \frac{\beta}{1 + \beta} \right) p_t^w BM_t L \quad (37)$$

$$p_t^w C_t^M = \left( \frac{1}{1 + \beta} \right) p_t^w BM_t L. \quad (38)$$

Thus, in each period  $t$ , the home economy exports  $\beta p_t^w BM_t L / (1 + \beta)$  units (in terms of agriculture) of manufacturing to the rest of the world in exchange for the same value of agriculture at the world price  $p_t^w$ . We characterize the BGP equilibrium of this regime in the following proposition:

**Proposition 3** (*The Strong SM<sub>0</sub> Case*). *Suppose that  $p_0^w > q_0^a$  and  $\bar{g} \geq g^w$ . If free trade is allowed for  $t \geq 0$ , (a) the economy is completely specialized in manufacturing, and this production pattern is sustainable, and (b) the free-trade BGP with complete specialization is a situation where  $C_t^M$ ,  $X_t^M$ , and  $M_t$  are growing at a constant rate of  $\bar{g} = \mu BL$ , while  $C_t^A$  is growing at a rate of  $\bar{g} - g^w$ . The economy exports  $BM_t L / (1 + \beta)$  units of the manufacturing goods and imports an equal value of agriculture.*

**Proof.** Since  $M_t$  is growing at the rate of  $\bar{g}$ , from  $X_t^M = BM_t L$  and (38), both  $X_t^M$  and  $C_t^M$  will also be growing at the same rate. In addition,  $p_t^w$  is declining at the given rate of  $g^w$ , condition (37) implies that  $C_t^A$  is growing at the rate of  $(\bar{g} - g^w)$ . Finally, the pattern of trade follows directly from the optimal consumption allocations (37) and (38). ■

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<sup>10</sup>Following the same argument adopted in the autarkic section, there cannot be any transition in this case.

### 3.2.2 The Weak $SM_0$ Case: $\bar{g} < g^w$

In this sub-case, because the world's price ratio is declining at a rate faster than that of the economy's MRT, there exists a time  $t = t^M$  so that  $p_t^w = q_t^f$ . See Figure 3.<sup>11</sup> At this point, the world's price line coincides with the economy's PPF. For  $t > t^M$ ,  $p_t^w < q_t^f$ , meaning that the economy has a comparative advantage in agriculture and exports the good. In other words, the economy's actual comparative advantage switches at  $t = t^M$ , and the initial specialization in manufacturing is not sustainable.

This result is similar to a result in Wong and Yip (1999), and indicates the fact that if the rest of the world grows faster than what the economy can potentially follow, specialization in manufacturing is not sustainable and the economy eventually will sooner or later turn to specialization in agriculture.

When the economy is completely specialized in agriculture, the case is similar to case  $SA_0$  and similar analysis can be applied here.

**Proposition 4** (*The Weak  $SM_0$  Case*). *Suppose that  $p_0^w > q_0^a$  and  $\bar{g} < g^w$ . If free trade is allowed for  $t \geq 0$ , the economy is initially completely specialized in manufacturing and exports the good. There exists a time  $t^M > 0$ , beyond which the economy exports agriculture.*

The above two propositions can be combined together to give the following corollary:<sup>12</sup>

**Corollary 1** *Suppose that  $p_0^w = q_0^a$  and that free trade is allowed for  $t \geq 0$ . The economy is completely specialized in the production of agriculture (manufacturing) if  $g^w > (<) g^a$ . The patterns of production and trade are sustainable.*

## 4 Dynamic Gains From Free Trade

Suppose that free trade is allowed for  $t \geq 0$ . We now examine whether the economy benefits dynamically. The two cases,  $SA_0$  and  $SM_0$ , are analyzed separately.

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<sup>11</sup>For convenience, the price and MRT in Figures 3 to 5 are drawn on a log scale so that a straight line represents a constant growth rate.

<sup>12</sup>In proving this proposition, it is noted that if  $g^w < g^a$ , then  $g^w < \bar{g}$ , meaning the pattern of production with an export of manufacturing is sustainable.

#### 4.1 Case $SA_0$ : $p_0^w < q_0^a$

Noting that free trade with complete specialization in agriculture is sustainable, substitute the BGP values of consumption given by conditions (35) and (36) into the welfare function in (6), which is then simplified to give

$$W^A = \frac{1}{\rho} \left[ (1 + \beta) \ln \left( \frac{\beta AL}{1 + \beta} \right) - \ln \beta - \ln p_0^w \right] + \frac{g^w}{\rho^2}. \quad (39)$$

We now compare this welfare function with that under autarky, which is given by (30). After simplification, we have

$$W^A - W^a = \frac{g^w - g^a}{\rho^2} + \frac{\ln p_0^a - \ln p_0^w}{\rho}. \quad (40)$$

The expression in (40) gives the gains from trade, which can be decomposed into two terms: the *growth effect*, which highlights the improvement in the growth rate of the consumption possibility frontier brought by the world, and the *dynamic terms of trade effect*, which comes from the difference between the world price ratio and the autarkic price ratio. Condition (40) implies that the economy gains from trade if and only if

$$\rho (\ln p_0^a - \ln p_0^w) > g^a - g^w. \quad (41)$$

Since in this case  $q_0^a = p_0^a > p_0^w$ , the LHS of (41) is positive. Depending on whether the economy is growing under autarky faster than the rest of the world, two sub-cases can be distinguished:

1. The Slow  $SA_0$  case:  $g^a \leq g^w$ . In this case, the RHS of (41) is negative, implying that condition (41) is satisfied. So free trade for  $t \geq 0$  is gainful.
2. The Fast  $SA_0$  case:  $g^a > g^w$ . In this case, the RHS of (41) is positive so that the condition may be violated. So free trade may harm the economy.

This result can be explained intuitively. In the Slow  $SA_0$  case, when free trade is first allowed, the economy instantaneously receives the static gains from trade. Under free trade, the consumption possibility frontier (CPF) of the economy shifts out at a rate equal to the world's growth rate  $g^w$ , which is



higher than the autarkic growth rate of the economy and the autarkic CPF. In other words, the economy is able to grow faster under free trade. Thus the economy is able to gain over time.

For the Fast  $SA_0$  case, under free trade (starting from  $t = 0$ ), the economy always has an actual comparative advantage in agriculture. However, if no trade is allowed instead, and because under autarky the economy grows faster than the ROW, its MRT will decline faster than the world price ratio so that there exists  $t = t^A$  at which  $q_t^a = p_t^w$ , as shown in Figure 4. After  $t^A$ , the economy has a potential comparative advantage in manufacturing. Because of the reversal of potential comparative advantage, the time space can be divided into two regions: region I for  $t \in [0, t^A)$ , and region II for  $t \geq t^A$ . In region I, the economy's actual comparative advantage coincides with its potential comparative advantage, and the economy is exporting the "right" commodity. So free trade tends to be gainful. In region II, the "wrong" good is exported since there is a conflict between the actual comparative advantage and the potential comparative advantage. As a result, the overall welfare impact of free trade that starts from  $t = 0$  is ambiguous.

**Proposition 5** (*Gains from Trade in Case  $SA_0$* ). *Suppose that  $p_0^w < q_0^a$  and that free trade exists starting from  $t = 0$ . Trade is gainful dynamically in the Slow  $SA_0$  case but not necessarily in the Fast  $SA_0$  case. A necessary and sufficient condition for a positive dynamic gain from trade is given by (41).*

## 4.2 Case $SM_0$ : $p_0^w > q_0^a$

In this case, the economy exports manufacturing under free trade at  $t = 0$ . The two sub-cases are analyzed separately.

### 4.2.1 The Strong $SM_0$ Case: $\bar{g} \geq g^w$

Since the export of manufacturing is sustained, the consumption of the two goods at time  $t$  is given by conditions (37) and (38). Substitute these values into the welfare function (6) and simplify the expression to give the intertemporal welfare of the economy:

$$W^M = \frac{1 + \beta}{\rho} \left[ \ln \left( \frac{\beta BL}{1 + \beta} \right) + \ln p_0^w + \ln M_0 \right] - \frac{1}{\rho} (\ln \beta + \ln p_0^w) + \frac{g^w}{\rho^2}. \quad (42)$$

Subtract the autarkic lifetime welfare in (30) from (42) to yield,

$$W^M - W^a = \frac{\beta(\ln p_0^w - \ln p_0^a)}{\rho} + \frac{\beta(\bar{g} - g^w) + (\bar{g} - g^a)}{\rho^2} > 0, \quad (43)$$

where the sign is based on the given conditions and the fact that  $\bar{g} > g^a$ . For convenience, we follow the notation introduced above and call the first term on the RHS of (43) the *dynamic terms of trade effect* and the second term the *growth effect*. The dynamic terms of trade effect is due to the economy's initial comparative advantage, while the growth effect comes from the growth gap. Note that the growth effect is still positive even if  $\bar{g} = g^w$ . Since both effects are positive, condition (43) implies that trade is gainful in this case. This result can be explained intuitively. As free trade is first allowed, the economy gets static gains from trade by exporting manufacturing, the good in which it has a comparative advantage. As the economy grows, its comparative advantage remains unchanged because its growth rate,  $\bar{g}$ , is not less than that of the world. There is no conflict between the economy's actual comparative advantage and the potential comparative advantage, and so the economy can gain from trade over time.

#### 4.2.2 The Weak $SM_0$ Case: $\bar{g} < g^w$

Since there is a switch in the economy's comparative advantage and the pattern of trade, its lifetime welfare can be obtained by using again (6) and the corresponding consumption derived earlier,

$$\begin{aligned} W^{MA} &= \int_0^{t^a} \left\{ (1 + \beta) \ln \left( \frac{\beta BL}{1 + \beta} p_t^w M_t^f \right) - \ln p_t^w \right\} e^{-\rho t} dt \\ &+ \int_{t^a}^{t^M} \left\{ (1 + \beta) \ln \left( \frac{\beta BL}{1 + \beta} p_t^w M_t^f \right) - \ln p_t^w \right\} e^{-\rho t} dt \\ &+ \int_{t^M}^{\infty} \left\{ (1 + \beta) \ln \left( \frac{\beta AL}{1 + \beta} \right) - \ln p_t^w \right\} e^{-\rho t} dt - \frac{\ln \beta}{\rho}. \quad (44) \end{aligned}$$

Subtracting the autarkic lifetime welfare given by (30) from (44) and rearranging the terms, we get

$$\begin{aligned}
W^{MA} - W^a &= \int_0^{t^a} \{(1 + \beta)(\bar{g} - g^a)t + \beta(\ln p_t^w - \ln p_t^a)\} e^{-\rho t} dt \\
&+ \int_{t^a}^{t^M} \{(1 + \beta)(\bar{g} - g^a)t + \beta(\ln p_t^w - \ln p_t^a)\} e^{-\rho t} dt \\
&+ \int_{t^M}^{\infty} \{(\ln p_t^a - \ln p_t^w)\} e^{-\rho t} dt. \tag{45}
\end{aligned}$$

It is easy to determine that the first and third terms on the RHS are positive while the sign of the second term is ambiguous. As a result, the sign of the overall welfare change is ambiguous. One sufficient condition for a positive gain is that  $\bar{g}$  is sufficiently close to  $g^a$ .

The intuition behind this result can be illustrated in Figure 5, which shows the paths of the world's price ratio  $p_t^w$ , the economy's MRT under autarky  $q_t^a$ , and the economy's MRT if free trade is allowed  $q_t^f$ .<sup>13</sup> The diagram gives two points of time,  $t = t^M$ , at which there is a switch in the actual comparative advantage of the economy, and  $t = t^a$ , at which there is a switch in the potential comparative advantage of the economy. Three regions can be identified, corresponding to the three terms in (45): (I)  $t \in [0, t^a)$  (II)  $t \in [t^a, t^M]$  and (III)  $t > t^M$ . In region I,  $q_t^a < p_t^w$ , and the economy is exporting manufacturing, the good in which it has an actual comparative advantage as well as potential comparative advantage. In region III,  $q_t^a > p_t^w$ , and with free trade starting from  $t = 0$ , the economy is exporting agriculture, also the "right" good. Therefore the economy gains in these two regions. In region II, however, there is a conflict between the actual comparative advantage (in manufacturing) and the potential comparative advantage (in agriculture). Thus trade in this region may be harmful.

**Proposition 6** (*Dynamic Gains from Trade in Case  $SM_0$* ). *Suppose that  $p_0^w > q_0^a$  and that free trade exists starting from  $t = 0$ . Trade is gainful in the Strong  $SM_0$  case but not necessarily in the Weak  $SM_0$  case. Trade is dynamically gainful if and only if the expression in condition (45) is positive. If it is further given that  $g^a$  is sufficiently close to  $\bar{g}$ , then the dynamic gain from trade is positive.*

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<sup>13</sup>This diagram is just Figure 3 with schedule  $q_t^a$  added. The relative slopes of the schedules are based on the condition  $g^w > \bar{g} > g^a$ .

The results in cases SA<sub>0</sub>(a) and SM<sub>0</sub>(a) can be combined to give the following proposition, the proof of which is straightforward and omitted here.

**Proposition 7** (*Dynamic Gains from Trade*). *Suppose that initially  $q_0^a = p_0^w$ . Then free trade starting from  $t = 0$  is beneficial. (In the singular case in which  $g^a = g^w$ , there is no change in the economy's welfare because no trade exists).*

## 5 Optimal Timing of Trade

In this section, we will answer the following two questions: First, is it good to delay the time when free trade is first allowed? Second, is it better to impose a production subsidy in addition to choosing the optimal timing of trade?

### 5.1 Free Trade

Define a new variable  $t_0 \geq 0$ , where the economy is under autarky for  $t \in [0, t_0)$  and free trade for  $t \geq t_0$ .<sup>14</sup> Treating  $t_0$  as a parameter, the lifetime welfare,  $\widetilde{W}$ , of the economy is equal to

$$\begin{aligned} \widetilde{W}(t_0) = & K_0 - (\ln p_0^a - \ln p_0^w) \int_0^{t_0} e^{-\rho t} dt - (g^a - g^w) \int_0^{t_0} t e^{-\rho t} dt \\ & + (1 + \beta)(\bar{g} - g^w) \int_{t_0}^{\infty} t e^{-\rho t} dt, \end{aligned} \quad (46)$$

where  $K_0 \equiv \{(1 + \beta) \ln[\beta AL / (1 + \beta)] - \ln \beta - \ln p_0^w\} / \rho - g^w / \rho^2$ . Note that  $p_0^a > p_0^w$  and  $\bar{g} > g^a > g^w$ ; so by (46)  $\widetilde{W}$  is decreasing in  $t_0$ . Denote the optimal value of  $t_0$  by  $\hat{t}_0$  that maximizes the intertemporal welfare of the economy. Condition (46) implies the following lemma:

**Lemma 1** If it is good to have free trade, it should be allowed as soon as possible, as long as there is no reversal in the pattern of trade.

It is shown earlier that in the Slow SA<sub>0</sub> and Strong SM<sub>0</sub> cases, free trade from  $t = 0$  is beneficial. Lemma 1 immediately implies that it is the best the

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<sup>14</sup>If  $t_0 = 0$ , it means that free trade is allowed from the beginning.

economy can do, meaning that in these two cases the optimal timing of free trade is  $\hat{t}_0 = 0$ .

In the other two cases, since free trade may not be beneficial, the optimal timing of free trade is not so clear. Let us first consider the Fast SA<sub>0</sub> case. Refer again to the two regions in Figure 4, where  $t = t^A < \infty$  is the time at which the economy's autarkic MRT equals the world's price ratio. If free trade is to start any time in region II, the economy exports manufacturing, and this pattern of trade is sustainable. Lemma 1 then implies that if free trade is allowed first in this region, it should be allowed as soon as possible. In other words within this region, the optimal value of  $t_0$  is  $t^A$ , with the resulting welfare equal to  $\widetilde{W}(t^A)$ . If, however, free trade is to be allowed some time in region I, it should be allowed as soon as possible. So the optimal timing of free trade in this region is  $t_0 = 0$ , with the corresponding welfare being equal to  $W^A$ . To determine when free trade should be allowed, the following rule gives the optimal timing,  $\hat{t}_0$ :

$$\hat{t}_0 = \begin{cases} 0 & \text{if } W^A \geq \widetilde{W}(t^A) \\ t^A & \text{if } W^A < \widetilde{W}(t^A). \end{cases} \quad (47)$$

Note that allowing autarky of the economy at all times is the same as setting  $t_0$  to be infinity. The rule in (47) implies that choosing  $t_0$  to be infinity is not optimal, meaning that if the economy can choose the optimal timing of free trade, it can do better than remaining under autarky at all time.

Similar analysis can be applied to the Weak SM<sub>0</sub> case. However, because the economy has the same potential comparative advantage (in agriculture) in regions II and III, the above analysis shows that if free trade is allowed from any time  $t \geq t^a$ , it should be allowed as soon as possible, i.e.,  $t^a$ . Thus we can focus on two possible actions of the government: (i)  $t_0 = 0$ , with the economy's welfare equal to  $W^M$ ; (ii)  $t_0 = t^a$ , with the welfare given by

$$\overline{W}(t^a) = \frac{1}{\rho} \left[ (1 + \beta) \ln \left( \frac{\beta AL}{1 + \beta} \right) - \ln \beta \right] - \int_0^{t^a} \ln p_t^a e^{-\rho t} dt - \int_{t^a}^{\infty} \ln p_t^w e^{-\rho t} dt.$$

To compare these two options, subtract  $\overline{W}(t^a)$  from the lifetime welfare  $W^M$  under free trade starting from  $t = 0$  to have

$$W^M - \overline{W}(t^a) = \frac{1}{\rho} (1 + \beta) (\ln p_0^w - \ln q_0^a) + \int_0^{t^a} (\ln p_t^a - \ln p_t^w) e^{-\rho t} dt. \quad (48)$$

Noting that  $q_0^a < p_0^w$  and  $g^w > \bar{g} > g^a$ , the welfare differential given by (48) has ambiguous sign. Therefore the rule for the optimal time for free trade is

$$\hat{t}_0 = \begin{cases} 0 & \text{if } W^M \geq \bar{W}(t^a) \\ t^a & \text{if } W^M < \bar{W}(t^a). \end{cases} \quad (49)$$

Using an argument similar to the one given earlier, it can be shown that free trade under rule (49) is better than no trade. The results obtained are summarized in the following proposition:

**Proposition 8** *In both the Slow SA<sub>0</sub> and Strong SM<sub>0</sub> cases, free trade should be allowed from the beginning. In the Fast SA<sub>0</sub> case, the optimal timing of free trade is given by condition (47), while in the Weak SM<sub>0</sub> case, the rule for optimal timing of free trade is given by condition (49). In each of these cases, free trade with an optimal timing is better than no trade.*

## 5.2 Trade with A Production Subsidy

We now analyze whether it is necessary to impose a production subsidy under trade. Let us begin with case SA<sub>0</sub>. As analyzed before, the economy will export agriculture, with no production in manufacturing, if free trade is allowed for  $t \geq 0$ . The question is, is it welfare improving to impose a production subsidy on manufacturing so that the economy produces and exports manufacturing? The advantage of this policy is that the economy is able to accumulate intangible capital, but the cost is that such protection is distortionary statically.

To answer the question, consider the following manufacturing subsidy:

$$\hat{s}_t = \begin{cases} q_t^f - p_t^w & \text{if } q_t^f > p_t^w \\ 0 & \text{if } q_t^f \leq p_t^w. \end{cases} \quad (50)$$

According to (50), the subsidy is imposed just big enough to have specialized production and export of manufacturing, but once the economy has achieved an actual comparative advantage in manufacturing the subsidy stops. The resulting lifetime welfare is equal to

$$\begin{aligned} W^{Ms} &= \frac{1}{\rho} \left[ (1 + \beta) \ln \left( \frac{\beta B L p_0^w M_0}{1 + \beta} \right) - \ln \beta - \ln p_0^w \right] \\ &+ \frac{(1 + \beta)(\bar{g} - g^w) + g^w}{\rho^2}. \end{aligned} \quad (51)$$

Subtract the autarkic welfare  $W^A$  from  $W^{Ms}$  to give

$$W^{Ms} - W^A = \frac{1 + \beta}{\rho} [\ln p_0^w - \ln q_0^a] + \frac{(1 + \beta)(\bar{g} - g^w)}{\rho^2}, \quad (52)$$

which is positive if and only if,

$$\rho [\ln p_0^w - \ln q_0^a] > g^w - \bar{g}, \quad (C)$$

i.e., the subsidy policy is a good one if and only if condition C is satisfied. Since the economy has an actual comparative advantage in agriculture at  $t = 0$ ,  $q_0^a > p_0^w$ , meaning that the LHS of (C) is negative. We immediately see that if  $\bar{g} \leq g^w$ , condition (C) is violated, meaning that no production subsidy should be imposed. This result is intuitive because if  $\bar{g} \leq g^w$ , then the economy can never have an actual comparative advantage in manufacturing and it never pays to subsidize the production of manufacturing.

We now turn to case  $SM_0$ . In the strong case, the export of manufacturing is sustainable. As a result, no production subsidy is necessary. In the weak case, there is a switch in the economy's potential comparative advantage. The question is whether it pays to subsidize the production of manufacturing after the switch in comparative advantage. The answer is no, because in this case  $\bar{g} < g^w$ , and as shown above, a production subsidy is not a good policy. The above results are summarized in the following proposition:

**Proposition 9** *Suppose that the government can choose the optimal manufacturing subsidy and the optimal timing of foreign trade. In case  $SA_0$ , free trade should start as soon as possible, and the optimal subsidy is given by (50) if and only if condition C is satisfied, or it is zero if  $g^w \geq \bar{g}$ . In case  $SM_0$ , no subsidy should be provided, while free trade should be allowed as soon as possible in the Strong case, or allowed at a time according to condition (49).*

The above proposition can be used to prove the following corollary:

**Corollary 2** If  $q_0^a = p_0^w$ , then the optimal production subsidy for the economy is zero, and free trade should start from  $t = 0$ , irrespective to the growth rates of the economy and the rest of the world.

The present analysis about subsidizing the sector that actual comparative advantage does not support is similar to the infant industry argument, but

there are some subtle differences.<sup>15</sup> The present necessary but not sufficient condition for subsidization,  $\bar{g} > g^w$ , is comparable to the Mill test for protection (Mill, 1848). As Bastable (1921) argued, the Mill test is not sufficient for a protection. Despite these similarities, some differences can be noted. First, the present analysis is a general equilibrium analysis, which provides a better picture of the opportunity cost of protection. Second, the present analysis allows the possibility that the subsidy may change the growth rate of the economy and that the world relative price of manufacturing is declining over time. Most work in analyzing the infant-industry argument assumes given world market conditions and the growth rate of the infant industry independent of the protective policy.<sup>16</sup> Third, in the present framework, a tariff does not work. Furthermore, in some cases, the optimal trade policy involves only the optimal timing of free trade, the optimal production subsidy being zero.<sup>17</sup> Fourth, modern theory of infant industry argument emphasizes the use of government intervention only in the presence of external distortions such as imperfect capital, technology spillover, and employment externality.<sup>18</sup> These externalities, if present, require appropriate policies to achieve an optimal equilibrium, but they usually stay when trade exists so that the corrective policy is needed forever. In the present model, dynamic externality exists in a closed economy. If in the presence of trade the optimal production subsidy is positive, it will be declining over time, and when the economy achieves a comparative advantage in manufacturing, no more subsidy is needed. That is a point when the dynamic externality has disappeared: Trade is able to eliminate the dynamic externality.

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<sup>15</sup>Earlier dynamic analyses of the infant industry argument include Clemhout and Wan (1970) and Bardhan (1971). See also Krugman (1984). However, the empirical evidence on the infant industry argument is mixed. Krueger and Tuncer (1982) conclude that using Turkish data the infant industry argument was not supported. Harrison (1984), however, casts doubt on the evidence against infant industry argument provided by Krueger and Tuncer.

<sup>16</sup>See, for example, Kemp (1960, Figure 1).

<sup>17</sup>Of course, even if a tariff works, it is generally not the first-best policy.

<sup>18</sup>If the growth of an infant industry is due to internal economies of scale and if the industry is worth supporting, then the firms will be willing to absorb the losses since the gains in the future will be able to cover the losses, meaning that no government intervention is needed.



## 6 Concluding Remarks

This paper introduced a model of a two-sector economy with the relative price of manufacturing declining at a constant rate over time. Manufacturing grows over time due to learning by doing. This promotes growth of the economy, but at the same time pulls down its relative price over time. The model was used to investigate several features of trade between the economy and the rest of the world; for example, whether the initial pattern of trade is sustainable over time.

The paper also analyzed whether free trade is gainful in a dynamic sense. Our analysis showed that free trade may lower the intertemporal welfare of the economy. This is not surprising because learning by doing is a source of dynamic externality. What is surprising is that if the government chooses the optimal timing of trade, then free trade is always beneficial in a dynamic sense. In other words, if optimal timing can be chosen, autarky is never the right policy.

This paper also investigates whether it pays to subsidize the production and export of manufacturing. This issue is relevant if the economy would export agriculture in the absence of any government intervention. The question is whether the government finds it beneficial to protect the manufacturing sector. This paper shows that there are cases in which such a protective policy makes sense, although the policy requires a declining production subsidy rate until the manufacturing sector can stand on its own.

The previous argument sounds similar to the traditional infant industry argument because both argue for protection. There are, however, several differences between the present argument and the infant industry argument. In the present model, there are cases in which free trade alone is enough to remove the dynamic externality. Even if production subsidy is needed, its rate will decrease over time until the manufacturing sector is strong enough to survive without any government protection. In the infant industry case, if government policy is needed because of external externalities such as imperfect capital market, technology spillover, and employment externality, then the externality will not go away under trade and a corrective policy is needed forever.

**Table 1: Summary of Results**

	Case SA <sub>0</sub>		Case SM <sub>0</sub>	
	Slow Case	Fast Case	Strong Case	Weak Case
features	$q_0^a > p_0^w,$ $g^a \leq g^w$	$q_0^a > p_0^w,$ $g^a > g^w$	$q_0^a < p_0^w,$ $\bar{g} \geq g^w$	$q_0^a < p_0^w,$ $\bar{g} < g^w$
trade pattern	exports A, sustainable	exports A, sustainable	exports M, sustainable	exports M, then A
gains from free trade	$W^A > W^a$	$W^A ? W^a$	$W^M > W^a$	$W^M ? W^a$
optimal timing, free trade	$\hat{t}_0 = 0$	$\hat{t}_0 = 0$ or $t^A$	$\hat{t}_0 = 0$	$\hat{t}_0 = 0$ or $t^a$
optimal timing, possible subsidy	$\hat{t}_0 = 0$ $\hat{s} = 0$ if condition C is violated* or if $\bar{g} < g^w$	$\hat{t}_0 = 0$ $\hat{s} = 0$ if condition C is violated*	$\hat{t}_0 = 0$ $\hat{s} = 0$	$\hat{t}_0 = 0$ or $t^a$ $\hat{s} = 0$

Note: \*If condition C is satisfied, the required production subsidy on sector M is  $\hat{s}_t = q_t^f - p_t^w$  when  $q_t^f \geq p_t^w$ , and  $\hat{s}_t = 0$  when  $q_t^f < p_t^w$ .

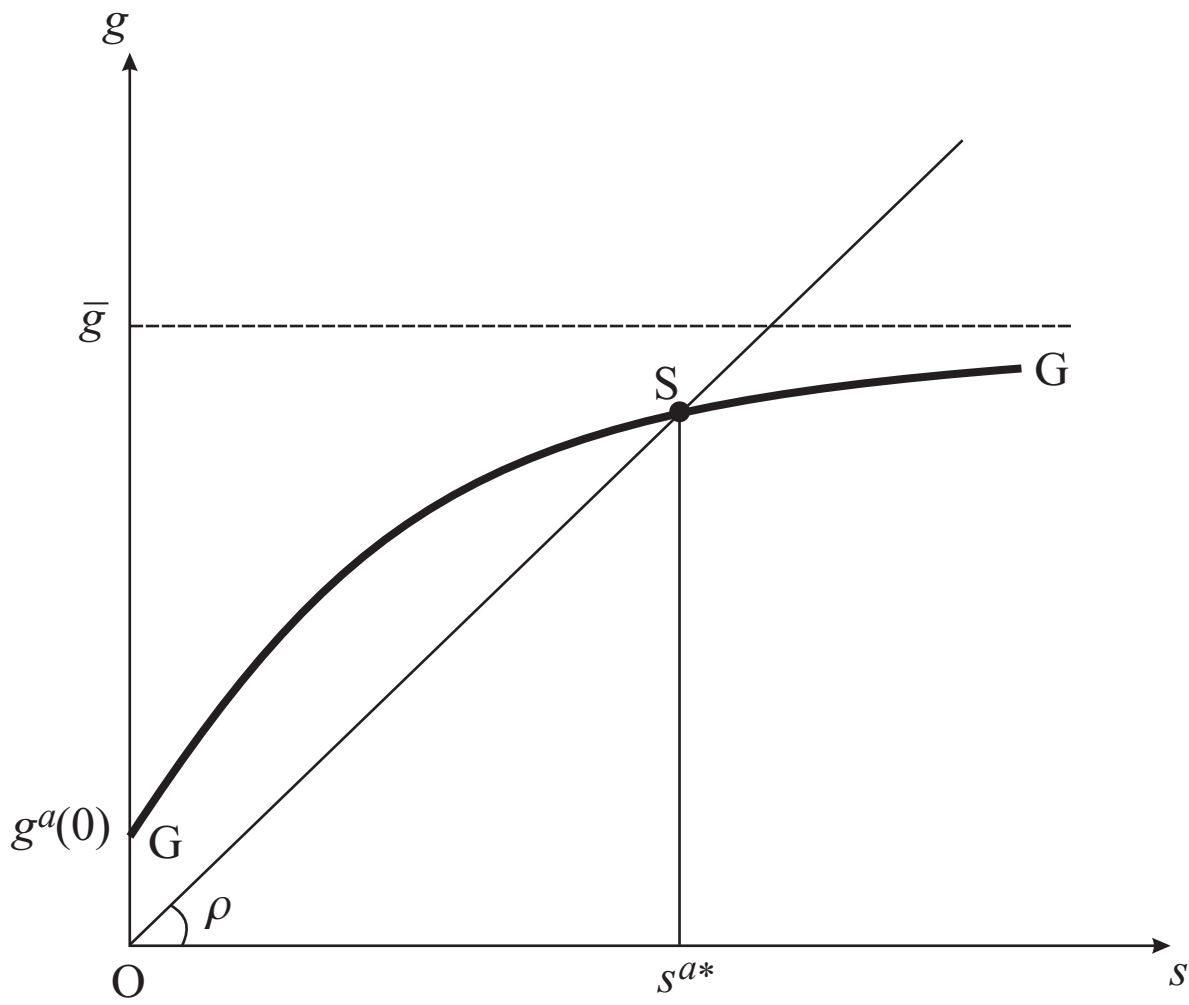


Figure 1

Growth Effect of a Production Subsidy

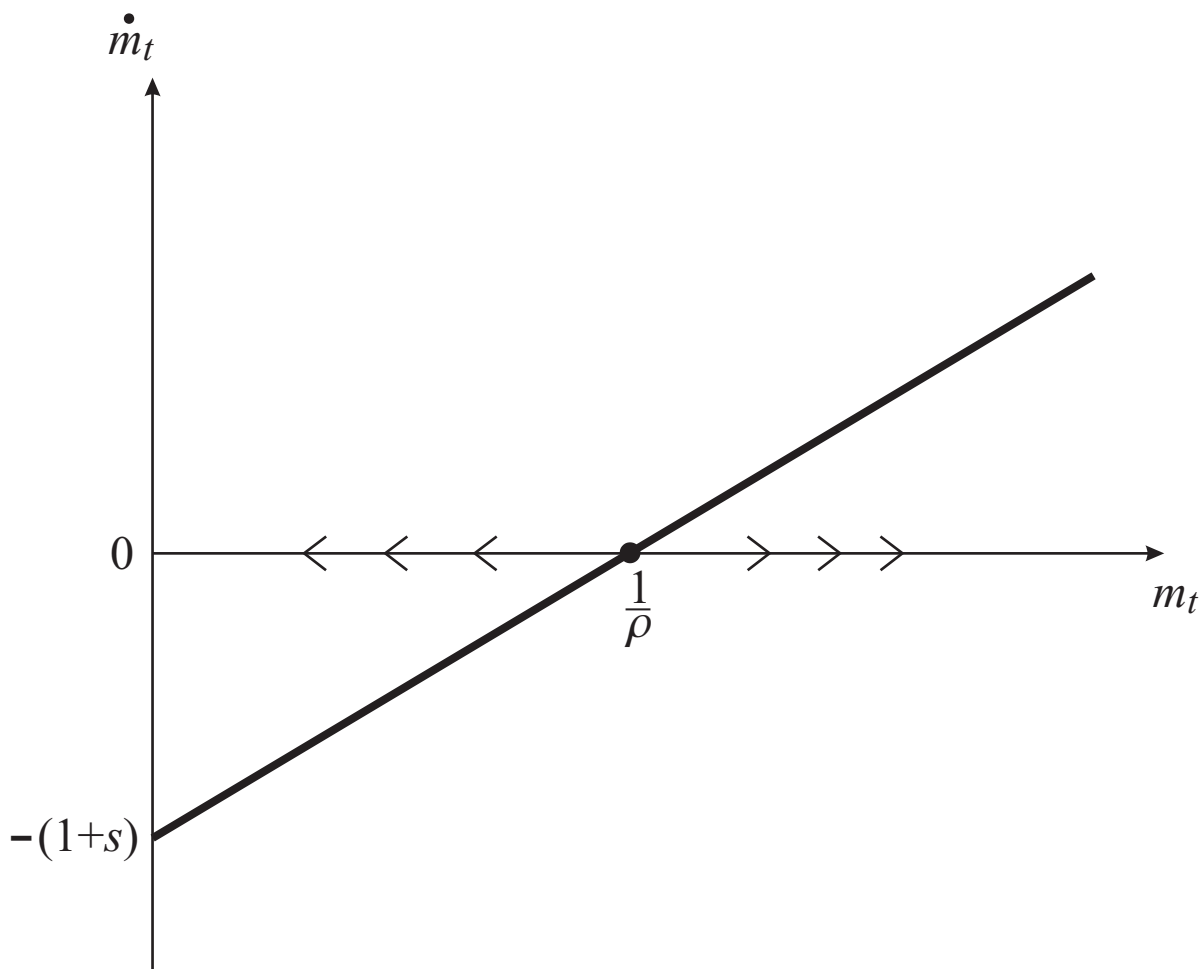


Figure 2

Instability of the Equilibrium in Terms of  $m_t$

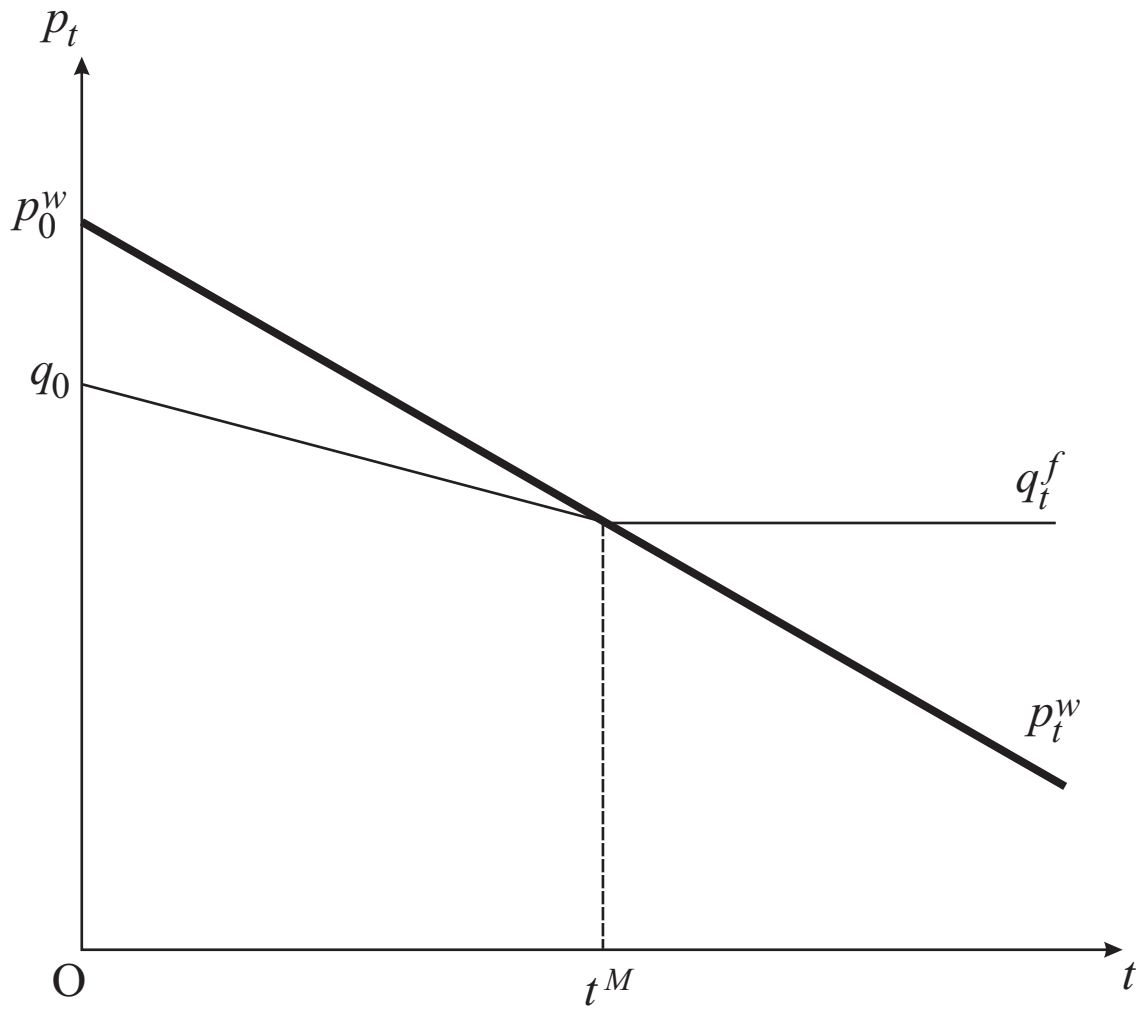


Figure 3

Pattern of Trade in the Weak  $SM_0$  Case

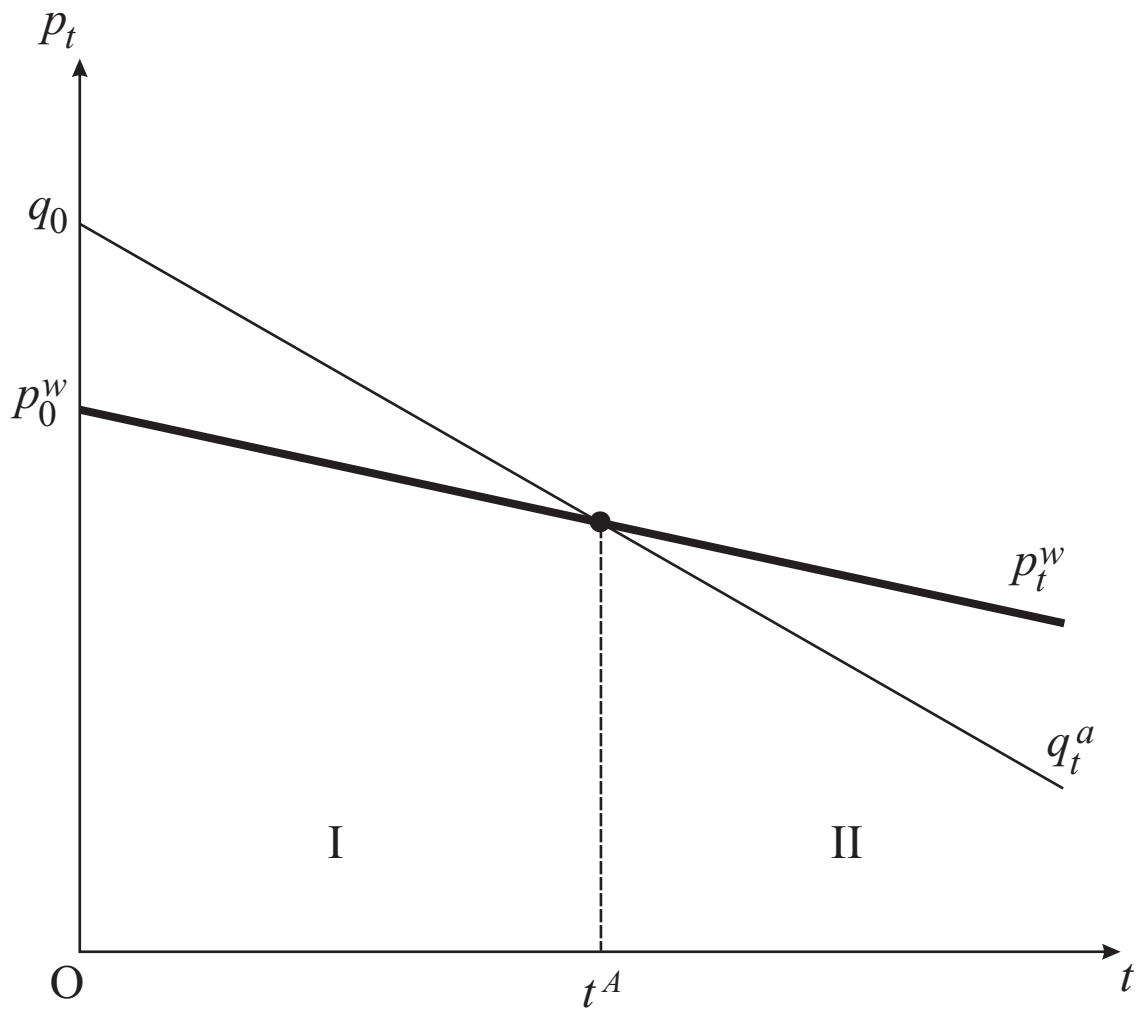


Figure 4

Pattern of Trade in the Slow  $SA_0$  Case

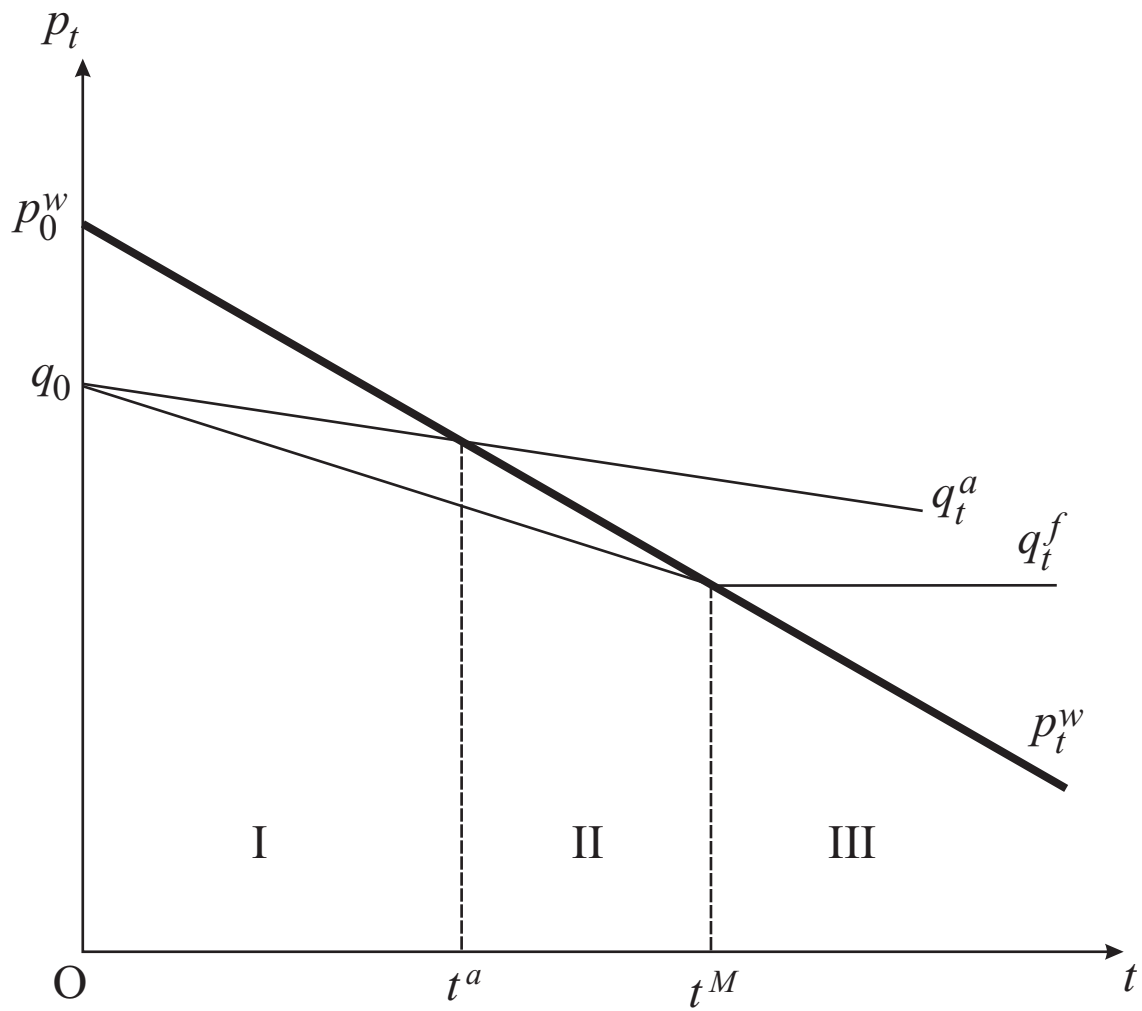


Figure 5

Autarkic PPF and Pattern of Trade in the Weak  $SM_0$  Case

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